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Application of Parseval's Theorem to Solve Definite Integrals with Maple

Chii-Huei Yu

School of Mathematics and Statistics, Zhaoqing University, Guangdong Province, China

Abstract: **This paper uses the mathematical software Maple as the auxiliary tool to study some type of definite integrals. We can obtain the infinite series form of this type of definite integral by Parseval's theorem. On the other hand, we provide an example to do calculation practically. The research method adopted in this study is to find solutions through manual calculations and verify these solutions using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking. For this reason, Maple provides insights and guidance regarding problem-solving methods.**

Keywords: **Definite integrals, Infinite series form, Parseval's theorem, Maple.**

I. INTRODUCTION

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. To get a better understanding of Maple can use the online system of Maple, or browsing Maple's website www.maplesoft.com. On the other hand, for the books on Maple can refer to [1-3].

In calculus and engineering mathematics, there are many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. In this article, we study the following type of definite integrals which is not easy to obtain its answer using the methods mentioned above.

$$\int_{-\pi}^{\pi} exp(rcos\theta)\cdot[\frac{1}{2}ln(1+r^2+2rcos\theta)\cdot cos(rsin\theta)+arctan\left(\frac{rsin\theta}{1+rcos\theta}\right)\cdot sin(rsin\theta)]d\theta, \quad (1)$$

where r,θ are real numbers, and $|r|<1$. The infinite series form of this type of definite integrals can be obtained by using Parseval's theorem; it is the main result of this paper (i.e., Theorem 1). Adams et al. [4], Nyblom [5], and Oster [6] studied some integral problems. Moreover, Yu [7-10] studied definite integral problems by Parseval's theorem. On the other hand, Yu [11,12] made use of another techniques to solve some types of definite integrals. Yu and Sheu [13] used area mean value theorem to solve some double integrals. In this study, we provide some examples to demonstrate the manual calculations, and verify the results by Maple.

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II. PRELIMINARIES AND RESULTS

Firstly, we introduce some definitions and formulas used in this paper.

2.1 Definitions.

2.1.1 Definition: Let $z = a + ib$ be a complex number, where $i = \sqrt{-1}$, and a, b are real numbers. a , the real part of z , is denoted by $\text{Re}(z)$; b , the imaginary part of z , is denoted by $\text{Im}(z)$. And the complex conjugate of z is denoted by $\bar{z} = a - ib$.

2.1.2 Definition: The complex logarithmic function $f(z) = lnz$ is defined by $lnz = ln|z| + i\theta$, where z is a non-zero complex number, θ is a real number, $z = |z| \cdot e^{i\theta}$, and $-\pi < \theta \leq \pi$.

2.2 Formulas.

2.2.1 Euler's formula

$exp(i\theta) = cos\theta + isin\theta$, where θ is a real number.

2.2.2 DeMoivre's formula

$(cos\theta + isin\theta)^p = cosp\theta + isinp\theta$, where p is an integer, and θ is a real number.

2.2.3 Suppose that z is a complex number, then the exponential function $exp(z) = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} z^n$.

2.2.4 If $|z| < 1$, then the logarithmic function $ln(1 + z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} z^n$.

2.2.5 Parseval's theorem

If $f(\theta)$ and$g(\theta)$ are two square integrable (with respect to Lebesgue measure), complex valued functions defined on R of period 2π with Fourier series expansions

$$f(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta}, g(\theta) = \sum_{n=-\infty}^{\infty} b_n e^{in\theta}$$

respectively, then $$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) \overline{g(\theta)} d\theta = \sum_{n=-\infty}^{\infty} a_n \overline{b_n}. \quad (2)$$

To obtain the major result, the following lemma is needed.

Lemma 1 *Assume that* r, θ *are real numbers, and* $1 + rcos\theta \neq 0$, *then the real part*

$$\text{Re}[ln(1 + re^{i\theta}) \overline{exp(re^{i\theta})}] = exp(rcos\theta) \cdot [\frac{1}{2} ln(1 + r^2 + 2rcos\theta) \cdot \cos(rsin\theta) + arctan\left(\frac{rsin\theta}{1+rcos\theta}\right) \cdot sin(rsin\theta)] .$$
(3)

Proof. $\text{Re}[ln(1 + re^{i\theta}) \overline{exp(re^{i\theta})}]$

$$= \text{Re}\{ln[(1 + rcos\theta) + irsin\theta)] \cdot exp(rcos\theta - irsin\theta)]$$

$$= \text{Re}\{\left(\frac{1}{2} ln(1 + r^2 + 2rcos\theta) + iarctan\left(\frac{rsin\theta}{1 + rcos\theta}\right)\right) \cdot exp(rcos\theta) \cdot [\cos(rsin\theta) - isin(rsin\theta)]\}$$

(by Definition 2.1.2 and Euler's formula)

$$= exp(rcos\theta) \cdot [\frac{1}{2} ln(1 + r^2 + 2rcos\theta) \cdot cos(rsin\theta) + arctan\left(\frac{rsin\theta}{1+rcos\theta}\right) \cdot sin(rsin\theta)] .$$

Q.e.d.

In the following, we use Parseval's theorem to determine the infinite series form of the definite integral (1).

Theorem 1 *Suppose that* r, θ *are real numbers, and* $|r| < 1$. *Then the definite integral*

$$\int_{-\pi}^{\pi} exp(rcos\theta) \cdot [\frac{1}{2} ln(1 + r^2 + 2rcos\theta) \cdot cos(rsin\theta) + arctan\left(\frac{rsin\theta}{1+rcos\theta}\right) \cdot sin(rsin\theta)] d\theta = 2\pi \cdot \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot n!} r^{2n}. \quad (4)$$

Proof. By Lemma 1, Parseval's theorem, Formulas 2.2.2, 2.2.3, and 2.2.4, we obtain

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} exp(rcos\theta)\cdot[\frac{1}{2}ln(1+r^2+2rcos\theta)\cdot cos(rsin\theta)+arctan\left(\frac{rsin\theta}{1+rcos\theta}\right)\cdot sin(rsin\theta)]d\theta=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\cdot n!}r^{2n}, \quad (5)$$

and hence the desired result holds. Q.e.d.

III. AN EXAMPLE

For the definite integral problem discussed in this study, an example is provided and we use Theorem 1 to obtain its infinite series form. Moreover, Maple is used to calculate the approximations of this definite integral and its infinite series form for verifying our answers.

Example 1. In Theorem 1, taking $r=-\frac{1}{3}$, then

$$\int_{-\pi}^{\pi} exp\left(-\frac{1}{3}cos\theta\right)\cdot\left[\frac{1}{2}ln\left(\frac{10}{9}-\frac{2}{3}cos\,\theta\right)\cdot cos\left(-\frac{1}{3}sin\theta\right)+arctan\left(\frac{-\frac{1}{3}sin\theta}{1-\frac{1}{3}cos\theta}\right)sin\left(-\frac{1}{3}sin\theta\right)\right]d\theta=$$

$$2\pi\cdot\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n\cdot n!}\left(-\frac{1}{3}\right)^{2n}. \quad (6)$$

We employ Maple to calculate the approximations of both sides of Eq. (6).

```
>evalf(int(exp(-1/3*cos(theta))*(1/2*ln(10/9-2/3*cos(theta))*cos(-1/3*sin(theta))+arctan(-1/3*sin(theta)/(1-1/3*cos(theta
)))*sin(-1/3*sin(theta))),theta=-Pi..Pi),18);
                    0.679208180934050879
>evalf(2*Pi*sum((-1)^(n-1)/(n*n!)*(-1/3)^(2*n),n=1..infinity),18);
                    0.679208180934050890
```

IV. CONCLUSION

From the discussion above, we know that Parseval's theorem is the main tool to find the infinite series form of some type of definite integrals. In fact, the applications of Parseval's theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and use Maple to verify our answers. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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